Sport League Scheduling

Jeffrey Larson
KTH Automatic Control Group
JANUARY 7, 2013

Contents

What are we trying to do?

Given a sports league decide, when and who plays in order to. . .

- satisfy some hard and/or soft constraints
- and possibly optimize something.

What are we trying to do?

Given a sports league decide, when and who plays in order to...

- satisfy some hard and/or soft constraints
- and possibly optimize something.

- English Premier League
  - 20 teams in a double round-robin tournament
- National Basketball Association
  - 30 teams each playing 82 games (2 Conferences, each with 3 divisions)
  - Each team plays:
    - 4 games against each team in their division (16 games)
    - 6 teams from other two divisions 4 times (24 games)
    - 4 teams from other two divisions 3 times (12 games)
    - All teams in the other conference twice (30 games)

Possible Objectives/Constraints

- Breaks
- Total Travel Distance
- Stadium Availabilities
- Adequate Team Rest
- Ease of Fan Travel
- Consecutive Meeting Between Teams
- Carry Over

Carry Over

	1	2	3	4	5	6	7
A	H	C	D	E	F	G	B
B	C	D	E	F	G	H	A
C	B	A	F	H	E	D	G
D	E	B	A	G	H	C	F
E	D	G	B	A	C	F	H
F	G	H	C	B	A	E	D
G	F	E	H	D	B	A	C
H	A	F	G	C	D	B	E

Carry Over

	A	B	C	D	E	F	G	H
A	0	0	3	0	1	2	1	0
B	5	0	0	0	1	0	0	1
C	0	1	0	3	0	3	0	0
D	0	2	0	0	2	0	3	0
E	1	1	0	2	0	2	0	1
F	0	0	0	0	2	0	3	2
G	0	3	1	0	0	0	0	3
H	1	0	3	2	1	0	0	0

Who Cares?

- Sports have a surprisingly large economic influence
- It's already going to happen, why not make it better?
- Many of the problems are NP-Hard.

NP-Hard

The decision problem:

> *"Can you feasibly schedule a round-robin tournament given a matrix of venue availabilities"*

is NP-Hard.

This is a fairly basic problem, so most often, alternative approaches are necessary.

Solution techniques

- IP formulation

Solution techniques

- IP formulation
  - Define binary variables x_{ijp} (1 if i hosts j in period p)

Solution techniques

- IP formulation
  - Define binary variables x_{ijp} (1 if i hosts j in period p)
  - Finding a feasible solution to this problem is NP-Hard

Solution techniques

- IP formulation
  - Define binary variables x_{ijp} (1 if i hosts j in period p)
  - Finding a feasible solution to this problem is NP-Hard
- First-Schedule-Then-Break

Solution techniques

- IP formulation
  - Define binary variables x_{ijp} (1 if i hosts j in period p)
  - Finding a feasible solution to this problem is NP-Hard
- First-Schedule-Then-Break
  - Assign who plays whom first, then decide who is the home team

Solution techniques

- IP formulation
  - Define binary variables x_{ijp} (1 if i hosts j in period p)
  - Finding a feasible solution to this problem is NP-Hard
- First-Schedule-Then-Break
  - Assign who plays whom first, then decide who is the home team
  - Finding a feasible solution to this problem is NP-Hard

Solution techniques

- IP formulation
  - Define binary variables x_{ijp} (1 if i hosts j in period p)
  - Finding a feasible solution to this problem is NP-Hard
- First-Schedule-Then-Break
  - Assign who plays whom first, then decide who is the home team
  - Finding a feasible solution to this problem is NP-Hard
- First-Break-Then-Schedule

Solution techniques

- IP formulation
  - Define binary variables x_{ijp} (1 if i hosts j in period p)
  - Finding a feasible solution to this problem is NP-Hard
- First-Schedule-Then-Break
  - Assign who plays whom first, then decide who is the home team
  - Finding a feasible solution to this problem is NP-Hard
- First-Break-Then-Schedule
  - Assign home/away patterns to each team, then schedule teams.

Solution techniques

- IP formulation
    - Define binary variables x_{ijp} (1 if i hosts j in period p)
    - Finding a feasible solution to this problem is NP-Hard
- First-Schedule-Then-Break
    - Assign who plays whom first, then decide who is the home team
    - Finding a feasible solution to this problem is NP-Hard
- First-Break-Then-Schedule
    - Assign home/away patterns to each team, then schedule teams.
    - Complexity unknown

Solution techniques

- IP formulation
  - Define binary variables x_{ijp} (1 if i hosts j in period p)
  - Finding a feasible solution to this problem is NP-Hard
- First-Schedule-Then-Break
  - Assign who plays whom first, then decide who is the home team
  - Finding a feasible solution to this problem is NP-Hard
- First-Break-Then-Schedule
  - Assign home/away patterns to each team, then schedule teams.
  - Complexity unknown
- Heuristics

Solution techniques

- IP formulation
  - Define binary variables x_{ijp} (1 if i hosts j in period p)
  - Finding a feasible solution to this problem is NP-Hard
- First-Schedule-Then-Break
  - Assign who plays whom first, then decide who is the home team
  - Finding a feasible solution to this problem is NP-Hard
- First-Break-Then-Schedule
  - Assign home/away patterns to each team, then schedule teams.
  - Complexity unknown
- Heuristics
  - Genetic Algorithms

Solution techniques

- IP formulation
  - Define binary variables x_{ijp} (1 if i hosts j in period p)
  - Finding a feasible solution to this problem is NP-Hard
- First-Schedule-Then-Break
  - Assign who plays whom first, then decide who is the home team
  - Finding a feasible solution to this problem is NP-Hard
- First-Break-Then-Schedule
  - Assign home/away patterns to each team, then schedule teams.
  - Complexity unknown
- Heuristics
  - Genetic Algorithms
  - Simulated Annealing

First-Break-Then-Schedule

Say we have a 6 team league, and the schedule requires

1. a round-robin tournament,
2. each team to play at home one of the last two periods,
3. no three straight away games allowed.

Then the possible HAPs are:

AAHHA	AHAHA
HAAHA	AHHHA
HAHHA	HHAHA
HHHHA	AAHAH
AHAAH	AHHAH
HAHAH	HHAAH
HHHAH	

Necessary Conditions

We can then take one HAP for each team to form a HAPSet:

Team 1	AHAHA
Team 2	AAHAH
Team 3	AHHAH
Team 4	HAHAH
Team 5	HHAHA
Team 6	HAAHA

When can a HAPSet be scheduled?

Necessary Conditions

We can then take one HAP for each team to form a HAPSet:

Team 1	AHAHA
Team 2	AAHAH
Team 3	AHHAH
Team 4	HAHAH
Team 5	HHAHA
Team 6	HAAHA

When can a HAPSet be scheduled?

A necessary condition is that, given any subset of teams, there must be "sufficient opportunities" for them to play each other.

Necessary Conditions

$$\sum_{p \in P} \min \left(c_A(T', p), c_H(T', p) \right) - \binom{|T'|}{2} \geq 0 \qquad \forall T' \subseteq T$$

Necessary Conditions

$$\sum_{p \in P} \min\left(c_A(T', p), c_H(T', p)\right) - \binom{|T'|}{2} \geq 0 \qquad \forall T' \subseteq T$$

Let $T' = \{1, 5, 6\}$.

p	$c_A(T', p)$	$c_H(T', p)$	min
1	1	2	1
2	1	2	1
3	3	0	0
4	0	3	0
5	3	0	0

Sufficient Conditions?

There are HAPSets which satisfy the previous necessary condition, but can not be scheduled.

Swedish Handball League

- 14 team league (consisting of two, 7-team pools).
- League play starts with a round-robin in the pool.
- Then a double round-robin tournament
- League wants teams to play HAH or AHA if they meet three times.